Database of Fully Developed Channel Flow

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## Contents

1 Comments
   1.1 Date of Release ........................................... 2
   1.2 Computers ................................................. 2
   1.3 Nomenclature ............................................... 2
   1.4 Description of Flow Field ................................. 2
   1.5 Flow conditions ........................................... 2
   1.6 Numerical Method ........................................... 3

2 Turbulence statistics ........................................... 5
   2.1 Mean ....................................................... 5
   2.2 Rms ....................................................... 8
   2.3 Skew ..................................................... 10
   2.4 Flat ..................................................... 12

3 Budgets of Reynolds Stresses and Turbulence Energy ....... 14
   3.1 Budget for $u\nu$ .......................................... 14
   3.2 Budget for $v\nu$ .......................................... 16
   3.3 Budget for $w\nu$ .......................................... 18
   3.4 Budget for $uv$ ........................................... 20
   3.5 Budget for $k$ ............................................ 22

4 One-dimensional Energy Spectra at several points ......... 24
   4.1 Streamwise Power Spectra ................................ 24
      4.1.1 at $y^+ = 5$ ........................................... 24
      4.1.2 at $y^+ = 15$ ......................................... 28
      4.1.3 at $y/\delta = 0.5$ ................................... 32
      4.1.4 at $y/\delta = 1$ ...................................... 36
   4.2 Spanwise Power Spectra .................................... 40
      4.2.1 at $y^+ = 5$ ........................................... 40
      4.2.2 at $y^+ = 15$ ......................................... 44
      4.2.3 at $y/\delta = 0.5$ ................................... 48
      4.2.4 at $y/\delta = 1$ ...................................... 52

5 Two-point Correlations at several points .................... 56
   5.1 Streamwise Separations .................................... 56
      5.1.1 at $y^+ = 5$ ........................................... 56
      5.1.2 at $y^+ = 15$ ......................................... 58
      5.1.3 at $y/\delta = 0.5$ ................................... 60
      5.1.4 at $y/\delta = 1$ ...................................... 62
   5.2 Spanwise Separations ....................................... 64
      5.2.1 at $y^+ = 5$ ........................................... 64
      5.2.2 at $y^+ = 15$ ......................................... 66
      5.2.3 at $y/\delta = 0.5$ ................................... 68
      5.2.4 at $y/\delta = 1$ ...................................... 70

6 Chebyshev Power Spectra ...................................... 72
1 Comments

1.1 Date of Release

June 11, 2002

1.2 Computers

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1.3 Nomenclature

\( \delta = \) channel half width
\( k = \) turbulent kinetic energy, \( u_i u_i / 2 \)
\( k_x, k_z = \) streamwise and spanwise wave numbers
\( \nu = \) kinematic viscosity
\( \omega_y = \) vorticity component in the \( y \)-direction
\( p = \) pressure
\( R(ab) = \) correlation coefficient between \( a \) and \( b \)
\( U_c = \) channel center mean velocity
\( U_m = \) bulk mean velocity
\( u, v, w = \) streamwise, wall-normal and spanwise velocity fluctuations
\( u_i = \) friction velocity
\( u_i = \) \( i \)-th component of velocity fluctuation
\( \gamma_i = u_i / u_\tau \), \( \nu / u_\tau \)
\( x, y, z = \) streamwise, wall-normal, spanwise directions
\( y^+ = y \times u_\tau / \nu \)

1.4 Description of Flow Field

The flow field simulated is a fully developed two-dimensional turbulent flow between two parallel walls. Hence, the flow is homogeneous both in the streamwise and spanwise directions and the statistics are dependent only upon the distance from the wall. The data presented here are non-dimensionalized by the wall variables, i.e., \( u_\tau \) and \( \nu \). The flow condition is defined by the pressure gradient imposed (or the friction velocity) and the distance between the walls (channel height).

1.5 Flow conditions

<table>
<thead>
<tr>
<th>Case</th>
<th>( Re_\tau = \delta \cdot u_\tau / \nu )</th>
<th>( Re_m = 2 \delta \cdot U_m / \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>109.429</td>
<td>3220.38</td>
</tr>
<tr>
<td>2</td>
<td>150.479</td>
<td>4586.21</td>
</tr>
<tr>
<td>3</td>
<td>297.899</td>
<td>10039.1</td>
</tr>
<tr>
<td>4</td>
<td>395.760</td>
<td>13924.9</td>
</tr>
<tr>
<td>5</td>
<td>642.540</td>
<td>24272.2</td>
</tr>
</tbody>
</table>
1.6 Numerical Method

Governing equations: a forth-order equation for $v$, a second-order equation for $\omega_y$ derived from the incompressible Navier-Stokes equations, and the continuity equation.

Discretization method: spectral method = $N_1 \times N_2$ Fourier series in the $x$- and $z$-directions, and Chebyshev polynomials up to the order $N_3$ in the $y$- direction (Chebyshev-tau method).

<table>
<thead>
<tr>
<th>Re$_r$</th>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$N_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>96</td>
<td>65</td>
<td>96</td>
</tr>
<tr>
<td>150</td>
<td>128</td>
<td>97</td>
<td>128</td>
</tr>
<tr>
<td>300</td>
<td>128</td>
<td>193</td>
<td>128</td>
</tr>
<tr>
<td>400</td>
<td>192</td>
<td>257</td>
<td>192</td>
</tr>
<tr>
<td>650</td>
<td>288</td>
<td>257</td>
<td>384</td>
</tr>
</tbody>
</table>

Aliasing treatment: nonlinear terms computed with collocation grids 1.5 times finer in the $x$- and $z$-directions, and 2 times finer in the $y$-direction.

Grid spacing: $\Delta x$ and $\Delta z$ viscous units in the $x$- and $z$-directions and $\Delta y_{min} \sim \Delta y_{max}$ in the $y$-direction.

<table>
<thead>
<tr>
<th>Re$_r$</th>
<th>$\Delta x^+$</th>
<th>$\Delta y_{min}^+$</th>
<th>$\Delta y_{max}^+$</th>
<th>$\Delta z^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>18.0</td>
<td>0.13</td>
<td>5.4</td>
<td>7.20</td>
</tr>
<tr>
<td>150</td>
<td>18.4</td>
<td>0.08</td>
<td>4.91</td>
<td>7.36</td>
</tr>
<tr>
<td>300</td>
<td>18.4</td>
<td>0.04</td>
<td>4.91</td>
<td>7.36</td>
</tr>
<tr>
<td>400</td>
<td>16.4</td>
<td>0.03</td>
<td>4.91</td>
<td>6.54</td>
</tr>
<tr>
<td>650</td>
<td>17.7</td>
<td>0.049</td>
<td>7.98</td>
<td>5.32</td>
</tr>
</tbody>
</table>

Time integration: A forth-order Runge-Kutta scheme for the nonlinear terms and a second-order Crank-Nicolson scheme for the viscous terms.

Size of computational box: $B_x \pi \delta \times 2 \delta \times B_z \pi \delta$.

<table>
<thead>
<tr>
<th>Re$_r$</th>
<th>$B_x$</th>
<th>$B_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>150</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>300</td>
<td>2.5</td>
<td>1</td>
</tr>
<tr>
<td>400</td>
<td>2.5</td>
<td>1</td>
</tr>
<tr>
<td>650</td>
<td>2.5</td>
<td>1</td>
</tr>
</tbody>
</table>

Initial conditions: $u$, $v$ and $u_{mean}$ given in accordance with the previous numerical results.

Criterion for stationary state: linear profile of total stress and stationary behavior of mean velocity and second-order moments of the three velocity components at some distance from the wall.

Length of time integration for ensemble averaging: about $T_1 \delta / u_+ (T_2 \nu / u_+^2)$ after the fully developed state is reached.

<table>
<thead>
<tr>
<th>Re$_r$</th>
<th>$T_1$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>2559</td>
<td>279991</td>
</tr>
<tr>
<td>150</td>
<td>2061</td>
<td>310130</td>
</tr>
<tr>
<td>300</td>
<td>277</td>
<td>82384</td>
</tr>
<tr>
<td>400</td>
<td>129</td>
<td>47364</td>
</tr>
<tr>
<td>650</td>
<td>40</td>
<td>25947</td>
</tr>
</tbody>
</table>

Error in continuity equation = $Max |div(\rho \cdot v)|$: not applicable
Courant number = $Max(\Delta t | u_i / \Delta x_i |) < 1.0$

Computer: HITACHI SR8000/128 ($C_1$ nodes (= $C_2$ cpus)) at the Computer Center of the University of Tokyo.
<table>
<thead>
<tr>
<th>$Re_\tau$</th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>150</td>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>300</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>400</td>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>650</td>
<td>8</td>
<td>64</td>
</tr>
</tbody>
</table>

Computation time: about $S_1$ sec actual computational time per one time step (= four sub-time steps).

<table>
<thead>
<tr>
<th>$Re_\tau$</th>
<th>$S_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>2.99</td>
</tr>
<tr>
<td>150</td>
<td>2.82</td>
</tr>
<tr>
<td>300</td>
<td>8.44</td>
</tr>
<tr>
<td>400</td>
<td>15.0</td>
</tr>
<tr>
<td>650</td>
<td>15.9</td>
</tr>
</tbody>
</table>

References

2 Turbulence statistics

2.1 Mean

![Graphs showing mean velocity profiles for different Reτ values.](image1.png)

![Graphs showing mean v- and w-velocity profiles for different Reτ values.](image2.png)
2.2 Rms
\[ \omega_{y,\text{rms}} \]

\[ \omega_{z,\text{rms}} \]

\[ p_{\text{rms}} \]
2.3 Skew
2.4 Flat
3 Budgets of Reynolds Stresses and Turbulence Energy

3.1 Budget for $u\varepsilon$
Visc Diff($u^+$)

Turb Diff($u^+$)

Diss($u^+$)

Res($u^+$)
3.2 Budget for $uv$

---

**ProdB$(v + v + )$**

---

**ProdB$(v + v + )$**

---

**$P_{Strain}(v + v + )$**

---

**Pres_Diff$(v + v + )$**

---

**Visc_Diff$(v + v + )$**

---
3.3 Budget for $ww$
3.4 Budget for $u v$

![Graphs showing budget for $u v$ at different Re values]
### 3.5 Budget for $k$

![Graphs showing budget for $k$ at different Reynold's numbers (Re)].

- **Prod**: Production of $k$.
- **Strain**: Strain rate contribution to $k$.
- **Pres**: Pressure diffusion term.
- **Visc**: Viscous diffusion term.

---

Each graph displays the contribution of each term to $k$ at different Reynold's numbers ($Re = 110, 150, 300, 400, 650$) across the boundary layer. The graphs illustrate how the production, strain, pressure diffusion, and viscous diffusion terms affect $k$, with each term depicted at different scales to highlight their contributions at various distances from the wall ($y^+$).
4 One-dimensional Energy Spectra at several points

4.1 Streamwise Power Spectra

4.1.1 at $y^+ = 5$

---

---
4.1.2 at $y^+ = 15$

\[ E(u^+ u^+)_{y^+ = 15} \]

\[ E(v^+ v^+)_{y^+ = 15} \]

\[ E(u^+ u^+)_{y^+ = 15} \]

\[ E(v^+ v^+)_{y^+ = 15} \]
4.1.3 at $y/\delta = 0.5$

\[ E(u^+ u^+) \delta = 0.5 \]

\[ E(v^+ v^+) \delta = 0.5 \]
- $Re_\tau = 110$  - - $Re_\tau = 300$  - $Re_\tau = 650$
- $Re_\tau = 150$  - - $Re_\tau = 400$

$E(w^+ w^+)_{y, \delta=0.5}$

$k_x \delta$

$10^{-2}$

$10^{-1}$

$10^{-5}$

$10^{-4}$

$10^{-3}$

$k_x^+$

$0.01$

$0.1$

$E(p^+ p^+)_{y, \delta=0.5}$

$k_x \delta$

$10^{-1}$

$10^{-2}$

$10^{-3}$

$10^{-4}$

$10^{-5}$

$k_x^+$

$0.01$

$0.1$
\[ E(-v^+ p^+_{y^{\delta=0.5}}) \]

- \( \text{Re}_\tau = 110 \)
- \( \text{Re}_\tau = 300 \)
- \( \text{Re}_\tau = 650 \)
- \( \text{Re}_\tau = 150 \)
- \( \text{Re}_\tau = 400 \)
4.1.4 at $y/\delta = 1$

\[ E(u^+ u^+)_{y/\delta = 1} \]

\[ E(v^+ v^+)_{y/\delta = 1} \]
\[ E(\nu^+ x)^{10^{-309}} \]

\[ k \delta \]

\[ k^+ \]

- \( Re_\tau = 110 \)
- \( Re_\tau = 300 \)
- \( Re_\tau = 650 \)
- \( Re_\tau = 150 \)
- \( Re_\tau = 400 \)
4.2 Spanwise Power Spectra

4.2.1 at $y^+ = 5$
4.2.2 at $y^+ = 15$

\[ E(u^+ u^+ y^+) = 15 \]

\[ E(v^+ v^+ y^+) = 15 \]
4.2.3 at $y/\delta = 0.5$
\[ E(u^+ v^+)_{y=0.5 \delta} \]

\[ k \delta \]

\[ k^+ \]

\[ E(u^+ p^+)_{y=0.5 \delta} \]

\[ k \delta \]

\[ k^+ \]

\( \text{Re}_\tau = 110 \quad \text{Re}_\tau = 300 \quad \text{Re}_\tau = 650 \)

\( \text{Re}_\tau = 150 \quad \text{Re}_\tau = 400 \)
\[ E(-v^+p^+) y^+_\delta = 0.5 \]

Graphs showing the variation of \( E(-v^+p^+) y^+_\delta = 0.5 \) with \( k \delta \) and \( k_z^+ \) for different \( \text{Re}_\tau \) values: 110, 300, 650, 150, and 400.
4.2.4 at $y/\delta = 1$

- $\text{Re}_\tau = 110$  
- $\text{Re}_\tau = 300$  
- $\text{Re}_\tau = 650$

- $\text{Re}_\tau = 150$  
- $\text{Re}_\tau = 400$

- $\text{Re}_\tau = 110$  
- $\text{Re}_\tau = 300$  
- $\text{Re}_\tau = 650$

- $\text{Re}_\tau = 150$  
- $\text{Re}_\tau = 400$
5 Two-point Correlations at several points

5.1 Streamwise Separations

5.1.1 at $y^+ = 5$
5.1.2 at $y^+ = 15$

\[ R(\text{uu}) y^+ = 15 \]

\[ R(\text{vv}) y^+ = 15 \]

\[ R(\text{ww}) y^+ = 15 \]

\[ R(\text{pp}) y^+ = 15 \]
5.1.3 at $y/\delta = 0.5$

- $R(uu)_{y/\delta=0.5}$
- $R(vv)_{y/\delta=0.5}$
- $R(ww)_{y/\delta=0.5}$
- $R(pp)_{y/\delta=0.5}$
5.1.4 at $y/\delta = 1$

\begin{align*}
R(uu)_{y/\delta=1} & \quad \Delta x^+ \\
R(vv)_{y/\delta=1} & \quad \Delta x^+ \\
R(ww)_{y/\delta=1} & \quad \Delta x^+ \\
R(pp)_{y/\delta=1} & \quad \Delta x^+ \\
\end{align*}
5.2 Spanwise Separations

5.2.1 at $y^+ = 5$
5.2.2 at $y^+ = 15$

\[ R(uu)_{y^+} = 15 \]

\[ R(vv)_{y^+} = 15 \]

\[ R(ww)_{y^+} = 15 \]

\[ R(pp)_{y^+} = 15 \]
5.2.3 at $y/\delta = 0.5$
5.2.4 at $y/\delta = 1$
6 Chebyshev Power Spectra

\begin{itemize}
\item $Re = 110$
\item $Re = 300$
\item $Re = 650$
\item $Re = 150$
\item $Re = 400$
\end{itemize}

\begin{itemize}
\item $Re = 110$
\item $Re = 300$
\item $Re = 650$
\item $Re = 150$
\item $Re = 400$
\end{itemize}