DNS of turbulent channel flow at $Re_{\tau} = 1160$ and evaluation of feedback control at practical Reynolds numbers

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In order to investigate the possibility of smart drag reduction in high Reynolds number wall turbulence, direct numerical simulation of a turbulent channel flow at $Re_{\tau} = 1160$ was made. The visualized flow field and the statistics suggest that not only the vortices but also the large-scale structures appear near the wall ($y^+ \sim 30$). Away from the wall, the low/high-speed large-scale structures prevail, and the vortices are clustered preferably in the low-speed ones. The contribution of the largescale structures to the Reynolds stress is much larger than that of the clustering vortices in the outer layer. Moreover, theoretical formulas are derived for the Reynolds number dependency on the drag reduction rate achieved by active feedback controls of wall turbulence, i.e., the opposition control (Choi *et al.*, 1994) and an ideal damping of the near-wall velocity fluctuations. The present formulas suggest that large drag reduction can be attained even at high Reynolds numbers if turbulence fluctuations adjacent to the wall are suppressed.

1 Introduction

Development of efficient turbulence control techniques for drag reduction and heat transfer augmentation is of great importance from the viewpoint of energy saving and environment impact mitigation. Among various methodologies, active feedback control schemes attract much attention because of their potential of large control effect with small power input (Moin and Bewley, 1994; Gad-el-Hak, 1996; Kasagi, 1998). The pioneering studies (Choi *et al.*, 1994; Lee *et al.*, 1998; Bewley *et al.*, 2001) have shown in their direct numerical simulation (DNS) of turbulent channel flow that the skin friction drag can be substantially reduced only by small amount of local blowing/suction on the wall.

However, the Reynolds numbers assumed in most previous studies are $Re_{\tau} = 100 - 180$ (hereafter, Re_{τ} denotes the friction Reynolds number defined by the wall friction velocity u_{τ} , the channel half-width δ , and the kinematic viscosity v), where significant low-Reynolds-number effects must exist. Iwamoto *et al.* (2002) showed in their DNS at $Re_{\tau} < 642$ that the control effect of the sub-optimal control (Lee *et al.*, 1998) is gradually deteriorated as the Reynolds number is increased. In real applications, in contrast, the Reynolds number is far beyond the values that DNS can handle. For a Boeing 747 aircraft, for example, the friction Reynolds number is roughly estimated to be $Re_{\tau} \sim 10^5$ under a typical cruising condition. For such high Reynolds number flows, where highly complex turbulent structures exist with a very wide range of turbulent spectra, no quantitative knowledge is available for predicting the effectiveness of active feedback control.

According to the analytical relation between the Reynolds shear stress distribution and the skin friction coefficient (Fukagata *et al.*, 2002), the amount of drag reduction depends on the suppression of Reynolds stress not only in the near-wall layer, but also away from the wall. As the Reynolds number increases, the contribution of the region away from the wall plays a dominant role (Iwamoto *et al.*, 2002). On the other hand, the basic strategy of the active feedback control using distributed sensors and actuators on the walls is the selective manipulation of the near-wall turbulence regeneration mechanism, and hence the suppression of turbulence intensity in the near-wall layer. Therefore, in general, it is not straightforward to predict the drag reduction rate at high Reynolds numbers, where the near-wall mechanisms have less contribution to the skin friction.

In the present study, direct numerical simulation of turbulent channel flow at moderate Reynolds number of $Re_{\tau} = 1160$ is made to examine the possibility of smart drag reduction in high Reynolds number wall turbulence. In addition, we theoretically investigate the Reynolds number effect on the drag reduction rate achieved by the opposition control (Choi *et al.*, 1994) and an idealized active feedback control with near-wall layer manipulation.

2 DNS of turbulent channel flow at $\text{Re}_{\tau} = 1160$

Up to now, various Reynolds number effects in wall turbulence have been reported. It is well known that near-wall vortices play an important role in the transport mechanisms in wall turbulence, at least, at low Reynolds number flows (Robinson, 1991). However, characteristics of near-wall coherent structures at higher Reynolds numbers still remain unresolved. Adrian *et al.*(2000) show that packets of large-scale hairpin vortices are often observed in high-Reynolds-number wall turbulence. In this section, the dynamics of the near-wall coherent structures are evaluated through DNS of turbulent channel flow at moderate Reynolds numbers.

The numerical method used in the present study is almost the same as that of Kim *et al.*(1987); a pseudo-spectral method with Fourier series is employed in the streamwise (*x*) and spanwise (*z*) directions, while a Chebyshev polynomial expansion is used in the wall-normal (*y*) direction. The Reynolds number Re_{τ} is 1160. Hereafter, *u*, *v*, and *w* denote the velocity components in the *x*-, *y*-, and *z*-directions, respectively. Superscript (+) represents quantities non-dimensionalized with u_{τ} and v.

Figure 1 shows a (x - z) plane view of an instantaneous partial flow field at $Re_{\tau} = 1160$, in which the vortices identified with isosurfaces of the second invariant of the deformation tensor $(Q^+ = -0.02)$ are visualized. It is found that the vortices form clusters in low-speed regions, and that some hairpin vortices are observed in high-speed regions.

Figure 2 shows a (y - z) cross-stream plane of an instantaneous partial flow field, in which contours of the streamwise velocity fluctuation u' and vortices ($Q^+ < -0.005$) are visualized, in order to examine the relationship between the near-wall vortices and the large-scale outer-layer structures. The near-wall vortices are located between low- and high-speed streaky structures as same as those in low Reynolds number flows (Kasagi *et al.*, 1995). Away from the wall, the



Figure 1: Plane view of vortices at $Re_{\tau} = 1160$. Iso-surface, $Q^+ = -0.02$; blue to red, $u'^+ = -1$ to $u'^+ = 1$.

low/high-speed large-scale structures prevail, and the vortices are clustered preferably in the low-speed regions. The streaky structures, of which spanwise spacing is about $100v/u_{\tau}$, exist only near the wall ($y^+ < 30$), while the large-scale structures exist from the center of the channel to the near-wall region ($y^+ \sim 30$). Note that the contribution of the large-scale structures to the Reynolds stress is much larger than that of the clustering vortices in the outer layer (not shown here).

3 Prediction of drag reduction rate by active feedback control

The decomposition of the skin friction drag τ_w proposed by Fukagata *et al.* (2002) reads

$$\tau_w^+ = \frac{3}{2} \frac{Re_b}{Re_\tau^2} + \frac{1}{\delta} \int_0^\delta 3\left(1 - \frac{y}{\delta}\right) \left(-\overline{u'^+ v'^+}\right) dy \tag{1}$$

in the wall unit of the uncontrolled flow. Here, the condition of constant flow rate is assumed (Re_b is the bulk Reynolds number), and the superscript of (+) denotes the wall unit of the uncontrolled flow. The overbar ($\overline{\cdot}$) and prime (\cdot') denote the mean and fluctuation components of the Reynolds decomposition. The first term on the RHS is the contribution of the laminar flow, while the second term is that of the turbulence, which is a weighted integral of the Reynolds shear stress distribution. As is noticed from Eq. (1), the difference in the areas covered by two (uncontrolled and controlled) curves of the weighted Reynolds stress directly corresponds to the drag reduction rate. In order to compute the drag reduction rate by using Eq. (1), we need to assume the Reynolds stress profiles of uncontrolled and controlled flows.

First, prediction of the drag reduction rate by the opposition control (Choi *et al.*, 1994) was made (Fukagata *et al.*, 2004). Here, we use a simple model based on the mixing length hypothesis with a damping function. For uncontrolled flow, the van Driest damping function is adopted to evaluate the mixing length. According to the previous studies (Choi *et al.*, 1994; Hammond *et al.*, 1998), the primary effect of the opposition control is likely to be the formation



Figure 2: Cross view of instantaneous velocity field at $Re_{\tau} = 1160$. Contours of the streamwise velocity fluctuation, blue to red, $u'^+ = -1$ to $u'^+ = 1$; white, $Q^+ < -0.005$.

of a virtual wall around $y^+ \simeq y_v^+/2$ (y_v is the position of the detection plane) which results in a thicker viscous sublayer than that of uncontrolled flow and an upward shift of velocity profile in the logarithmic region. A relatively universal behavior of the mixing length is also observed in the simulation of controlled channel flow at $Re_{\tau} = 110 - 642$ with the detection plane at $y^+ = 10$ (Iwamoto *et al.*, 2002). Therefore, we simply assume the mixing length for the controlled flow from curve fitting of DNS data (not shown here). Once the mixing length is modeled as above, the Reynolds stress can easily be computed by using the classical theories.

The drag reduction rate, R_D , predicted by the procedure described above is shown in Fig. 3 as a function of Re_{τ} . In the high Reynolds number region, say $Re_{\tau} > 10^3$, the predicted drag reduction rate gradually decreases with the increase of Reynolds number. This is associated with the dominance of the contribution from the Reynolds stress away from the wall (Iwamoto *et al.*, 2002). On the other hand, the increasing behavior in the low Reynolds number region ($Re_{\tau} \sim 10^2$) is attributed to the increase of control margin as is suggested by Eq. (1). The dependency of the drag reduction rate to the Reynolds number is relatively weak, despite the decrease of relative importance of the near-wall layer that can be directly manipulated. This weak Reynolds number dependency is due to the secondary effect (Fukagata and Kasagi, 2003). Namely, the Reynolds shear stress in the region far-from the wall is indirectly suppressed following the direct suppression in the near-wall layer.

Next, we theoretically investigate the Reynolds number effect on the drag reduction rate achieved by an idealized active feedback control with near-wall layer manipulation (Iwamoto *et al.*, 2004). We assume that all velocity fluctuations in the near-wall layer, i.e., $0 \le y \le y_d$, are perfectly damped. We also assume fully-developed turbulent channel flows under a constant



Figure 3: Predicted drag reduction rate by the opposition control. Solid lines, channel flow; dotted lines, pipe flow; marks, DNS data (Hammond *et al.*, 1998; Iwamoto *et al.*, 2002; Fukagata and Kasagi, 2003).

flow rate. We can derive a theoretical formula between the Reynolds number of the uncontrolled flow Re_{τ} , the thickness of damping layer divided by channel half-width y_d/δ , and the drag reduction rate R_D , only by adopting Dean's formula (1978) derived from the logarithmic law.

Figure 4 shows the dependency of R_D on Re_{τ} for constant values of y_d . As Re_{τ} increases, R_D decreases. The Reynolds number dependency of R_D , however, is found to be very mild. For $y_d^+ = 10$, for instance, the drag reduction rate R_D is about 43% at $Re_{\tau} = 10^3$, and about 35% even at $Re_{\tau} = 10^5$. The damping layer in the latter case is extremely thin as compared to the channel half width, i.e., $y_d/\delta = 0.01\%$.

As is noticed from Eq. (1), the difference in the areas surrounded by the two (uncontrolled and damped) curves of the weighted Reynolds stress directly corresponds to the drag reduction rate. For higher Reynolds numbers, the relative thickness of the damping layer y_d/δ becomes smaller so that the contribution away from the damped layers should be dominant. Thus, large drag reduction by the wall control at high Reynolds numbers is mainly attributed to the decrease of the Reynolds stress in the region away from the wall.

4 Conclusions

Direct numerical simulation of a turbulent channel flow at $Re_{\tau} = 1160$ was made in order to examine the relationship between the near-wall vortices and the large-scale outer-layer structures. The vortices are clustered in the low-speed large-scale structures, however, the contribution of the vortices to the Reynolds stress is much less than that of the large-scale structures in the outer layer.

We also derived formulas to describe the relationship between the Reynolds number and the drag reduction rate in turbulent channel flows, by assuming the opposition control (Choi *et al.*, 1994) and an ideal damping of the velocity fluctuations in the near-wall layer. The derived formula indicates that large drag reduction can be attained even at high Reynolds numbers by



Figure 4: Dependency of the drag reduction rate R_D on the Reynolds number Re_{τ} with the constant thickness of damping layer y_d .

suppressing the turbulence only near the wall, viz., without any direct manipulation of largescale structures away from the wall. Therefore, the basic strategy behind the existing control schemes, i.e., attenuation of the near-wall turbulence only, is also valid at very high Reynolds numbers appearing in real applications.

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