On the lower bound of net driving power in controlled duct flows

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Abstract

We examine mathematically the lower bound of the net driving power (i.e., the summation of pumping and actuation powers) of a controlled duct flow under a constant flow rate. The net power in a duct with arbitrary cross-section in the presence of the inertial term, blowing/suction from the wall, and arbitrary body forces can be decomposed into four terms: (1) dissipation due to the velocity profile of Stokes flow (in other words, pumping power for the Stokes flow); (2) dissipation due to deviation of mean velocity from the Stokes flow profile; (3) dissipation due to velocity fluctuations; and (4) correlation between the wall-pressure of Stokes flow and the time-averaged blowing/suction velocity. Among these, the first three terms are shown to be non-negative, while the sign of the fourth term is indefinite. The fourth term vanishes in the cases where the duct has a constant-shape cross-section, such as cylindrical pipes and plane channels. Namely, in such cases, the lower bound of net power is exactly given by the dissipation rate (pumping power) of the Stokes flow at the same flow rate.

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1 Introduction

The skin friction drag of a wall-bounded turbulent flow is usually much larger than that of a laminar flow under the same bulk Reynolds number due to the augmented momentum transfer. Since 1990's, various feedback control schemes have been proposed for friction drag reduction in wall-bounded turbulent flows and examined by means of direct numerical simulation (DNS) and wind-tunnel experiments, as is introduced in the recent review papers [1–4]. Among others, Bewley et al. [5] demonstrated that a low Reynolds number turbulent channel flow can be re-laminarized by applying the optimal control technique.

For the turbulence control aiming at friction drag reduction, it is essential to know its theoretical limitations. Concerning the lower bound of skin friction drag, Bewley [1] made the following conjecture (hereafter referred to as Bewley's conjecture):

The lowest sustainable drag of an incompressible constant mass-flux channel flow, when controlled via a distribution of zero-net mass-flux blowing/suction over the no-slip channel walls, is exactly that of the laminar flow.

On the other hand, Fukagata et al. [6] derived an identity equation between the Reynolds shear stress and the skin friction coefficient, $C_f = 2\tau_w^*/(\rho^* U_b^{*2})$ (where τ_w^* is the wall shear, ρ^* is the density, U_b^* is the bulk-mean velocity, and the superscript of $*$ denotes dimensional quantities). For fully-developed flows in a cylindrical pipe and a plane channel, this identity reads

$$C_f = \frac{16}{Re_b} + 32\int_0^1 (\overline{u_r' u_z'}) r^2 dr \quad \text{(pipe)} \tag{1}$$

and

$$C_f = \frac{12}{Re_b} + 12\int_{-1}^1 (\overline{u'v'}) y dy \quad \text{(channel)}, \tag{2}$$

respectively. The coordinate systems are defined as shown in Figs. 1(a) and (b) and all the quantities are made dimensionless by using twice the bulk mean velocity, $2U_b^*$, the pipe radius/channel half-width, δ^*, and the density, ρ^*. The bulk Reynolds number, Re_b, is defined as

$$Re_b = \frac{2U_b^*\delta^*}{\nu^*}\ , \tag{3}$$

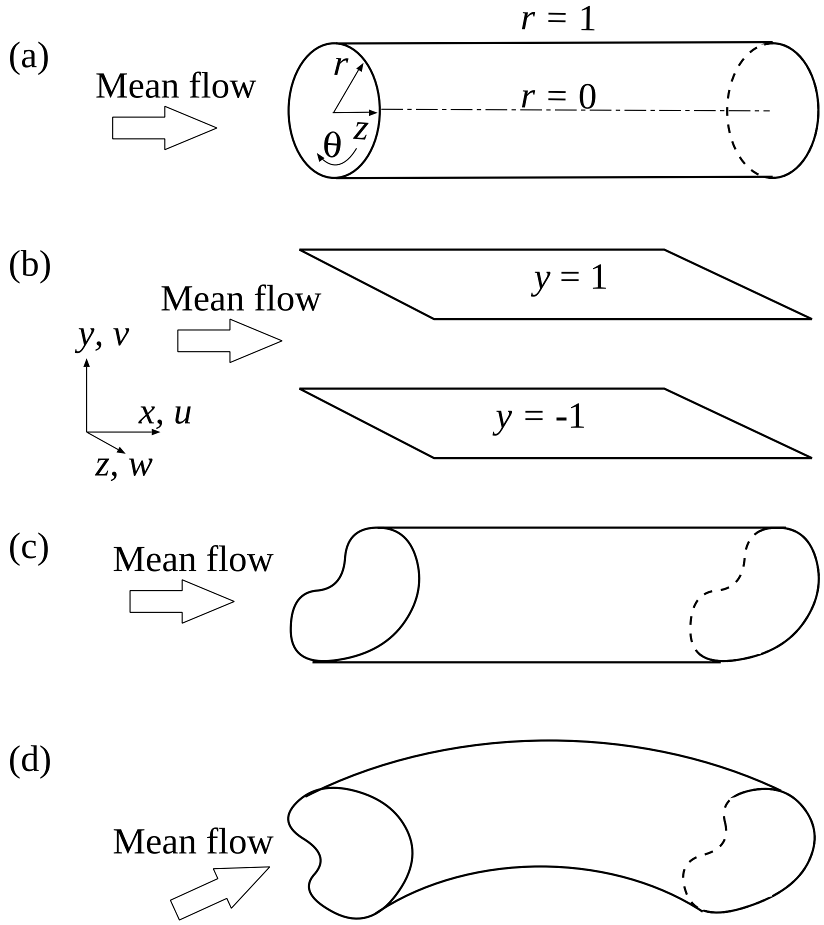

Fig. 1. Geometry of ducts with constant-shape cross-section, to which the lower bound of net driving power is provided: (a) pipe; (b) channel; (c) straight duct; (d) duct with constant streamline curvature.

where ν^* is the kinematic viscosity. The overbar $(\bar{\cdot})$ denotes the average in the streamwise and azimuthal/spanwise directions as well as in time, and the prime $(\cdot')$ denotes the corresponding fluctuation components. The first terms in the right-hand-side (RHS) of Eqs. (1) and (2) are identical to the laminar friction drag. The second terms are the turbulent contribution, which is expressed by a weighted integration of the Reynolds shear stress. Note that a similar identity has been presented also by Bewley and Aamo [7] and a generalized expression has been derived by Sbragaglia and Sugiyama [8]. Moreover, the identity has been extended to various flow systems, e.g., polymer/surfactant-added channel/boundary layer flows [9, 10] and a bubble-added Taylor-Couette flow [11].

Equations (1) and (2) imply that, against Bewley's conjecture, a sublaminar friction drag (i.e., friction drag lower than that of the laminar flow) can be attained if the second term can be made negative [6]. Following this implication, some counter examples to Bewley's conjecture have been reported. Fukagata et al. [12] applied a virtual feedback force in their DNS of pipe flow so as to change the sign of

Reynolds shear stress in the region near the wall and attained a sublaminar friction drag. More recently, Min et al. [13] demonstrated by a linear analysis as well as by DNS that the sublaminar friction drag is made possible by applying an upstream traveling wave-like blowing and suction from the channel walls. Marusic et al. [14] further discussed the conditions on which such sublaminar drag can be attained by blowing/suction control.

Of more engineering importance, however, is the theoretical limitation on the net power required to drive a flow at a given flow rate when the flow is actively controlled. The net power means the summation of the pumping power, W_p, and the power required for actuation, W_a. Now, we may draw a conjecture similar to Bewley's, but for the net power:

> "The lowest net power required to drive an incompressible constant mass-flux channel flow, when controlled via a distribution of zero-net mass-flux blowing/suction over the no-slip channel walls, is exactly that of the laminar flow."

To the authors' knowledge, neither a mathematical proof has been given nor a counter example has been reported to this conjecture. Although there is a similar mathematical argument called Helmholtz and Korteweg theorem [15], it is based on the assumption that the inertial term can be neglected and, of course, does not account for any active control input. The lack of mathematical proof to this conjecture can also be noticed from the statement in the very recent review by Kim and Bewley [3], which says that the first fundamental limitation established for Navier-Stokes equations is the minimum heat transfer of a channel flow [16].

In the present paper, we give a mathematical proof of the abovementioned conjecture in a generalized form. Namely, we derive the following theorem:

> "The lowest net power required to drive an incompressible constant mass-flux flow in a periodic duct having arbitrary constant-shape cross-section, when controlled via a distribution of zero-net mass-flux blowing/suction over the no-slip channel walls or via any body forces, is exactly that of the Stokes flow."

In order to derive this theorem, we first derive the global energy balance in an arbitrary periodic duct. Then, as special cases of the energy balance, we show the lower bound of net power for the ducts with constant-shape cross-section exemplified in Fig. 1.

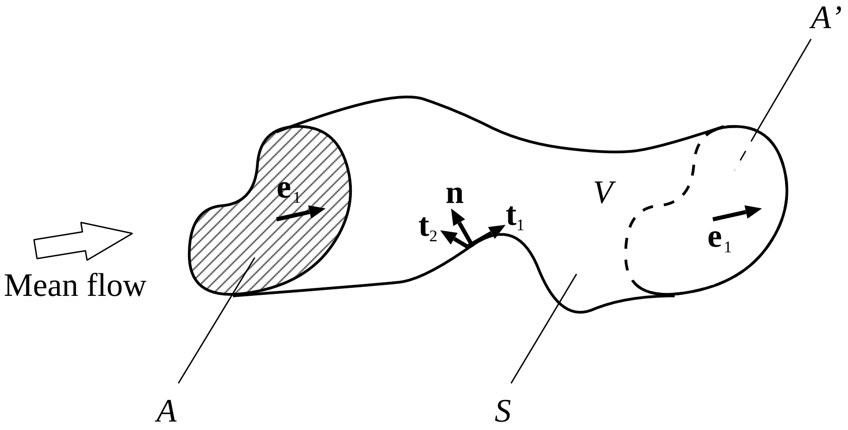

Fig. 2. Geometry of an arbitrary periodic duct considered for the derivation of energy balance equation and the definitions of volume (V), cross-sections (A and A'), wall surface (S), unit vector normal to cross-sections ($\mathbf{e}_1$), unit vectors normal ($\mathbf{n}$) and tangential to the wall ($\mathbf{t}_1$ and $\mathbf{t}_2$).

2 Global energy balance of a controlled flow in an arbitrary periodic duct

We consider a fully-developed isothermal incompressible flow in a duct with arbitrary cross-section. The fluid is assumed to be Newtonian and have constant physical properties. The duct is assumed to have a periodicity at a finite streamwise length, as shown in Fig. 2. The flow is driven by a time-dependent external pressure gradient that is adjusted to keep the flow rate constant. The continuity and momentum equations are expressed in the vector form as

$$\nabla \cdot \mathbf{u} = 0 \tag{4}$$

and

$$\frac{\partial \mathbf{u}}{\partial t} = \nabla \cdot \left[-\mathbf{u}\mathbf{u} - p\,\mathbf{I} + \frac{2}{Re_b}\mathbf{s} \right] + \mathbf{b} \,, \tag{5}$$

where $\mathbf{u}$ is the velocity vector, p is the pressure, and $\mathbf{I}$ is the unit dyadic, respectively. The strain rate tensor, $\mathbf{s}$, is defined as

$$\mathbf{s} = \frac{1}{2}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right] \,, \tag{6}$$

where the superscript of T denotes the transpose. All the quantities are made dimensionless by using twice the bulk mean velocity, $2U_b^*$, the unit length, δ^* (which can be arbitrarily chosen depending on the geometry) and the density, ρ^*. The bulk Reynolds number, Re_b, is defined by Eq. (3). Any body force used for actuation is denoted by **b**. The wall boundary condition allows zero-net-flux blowing and suction, while the no-slip condition is applied in the tangential directions, i.e.,

$$\mathbf{u}_{wall} = \phi \mathbf{n} \, , \tag{7}$$

with

$$\int_S \phi \, dS = 0 \, , \tag{8}$$

where **n** and S denote the unit wall-normal vector (directed toward the fluid side) and the wall surface area, respectively. The condition of constant flow rate is expressed as

$$\int_A \mathbf{u} \cdot \mathbf{e}_1 \, dA = AU_b \, , \tag{9}$$

where U_b is the bulk-mean velocity and $\mathbf{e}_1$ denotes the unit vector normal to the cross-section A. The flow field is statistically periodic in the streamwise direction, i.e.,

$$\overline{f}(\mathbf{x} \in A') = \overline{f}(\mathbf{x} \in A) \, , \tag{10}$$

where f represents products of $\mathbf{u}$ and p' of any orders (i.e., $\mathbf{u}$, p', $(\mathbf{u} \cdot \mathbf{u})$, $(p'\mathbf{u})$, etc.), and A and A' formally denote the upstream and downstream cross-sections of the periodic duct, respectively. Hereafter, the overbar $(\bar{\cdot})$ to a flow variable is redefined as the temporal average (with an infinitely long time span) and the corresponding fluctuation component is denoted by a prime $(\cdot')$. For the quantities concerning the mean pressure, the statistical periodicity reads

$$\begin{aligned} \int_{A'} \overline{p} \, dA' &= \int_A \overline{p} \, dA - A\Delta P \\ \int_{A'} \overline{p}\mathbf{u} \cdot \mathbf{e}_1 \, dA' &= \int_A \overline{p}\mathbf{u} \cdot \mathbf{e}_1 \, dA - AU_b\Delta P \, , \end{aligned} \tag{11}$$

where ΔP denotes the pressure drop in the periodic length.

First, we introduce some mathematical relationships that are used in the derivation process. On the wall, the following identities hold for the normal components of the deformation rate tensor:

$$\mathbf{t}_1 \cdot \mathbf{s} \cdot \mathbf{t}_1 = \underbrace{\frac{\partial(\mathbf{u}\cdot\mathbf{t}_1)}{\partial t_1}}_{=0} - \kappa_1 \mathbf{u}\cdot\mathbf{n} = -\kappa_1 \phi \,, \tag{12}$$

$$\mathbf{t}_2 \cdot \mathbf{s} \cdot \mathbf{t}_2 = \underbrace{\frac{\partial(\mathbf{u}\cdot\mathbf{t}_2)}{\partial t_2}}_{=0} - \kappa_2 \mathbf{u}\cdot\mathbf{n} = -\kappa_2 \phi \,, \tag{13}$$

and

$$\mathbf{n}\cdot\mathbf{s}\cdot\mathbf{n} = \underbrace{\nabla\cdot\mathbf{u}}_{=0} - \mathbf{t}_1\cdot\mathbf{s}\cdot\mathbf{t}_1 - \mathbf{t}_2\cdot\mathbf{s}\cdot\mathbf{t}_2 = (\kappa_1+\kappa_2)\phi = -(\nabla\cdot\mathbf{n})\phi \,, \tag{14}$$

where $\mathbf{t}_1$ and $\mathbf{t}_2$ denote the unit vectors tangential to the wall, as shown in Fig. 2, which are the principal directions corresponding to the principal curvatures, κ_1 and κ_2, respectively. We should stress again that the wall-normal unit vector $\mathbf{n}$ is taken in the direction from wall- to fluid side, so that the relationship between the curvature and $\mathbf{n}$ is given by $(\kappa_1+\kappa_2) = -(\nabla\cdot\mathbf{n})$. For the scalar product of $\mathbf{s}$ and $\nabla\mathbf{u}$, the following identity holds when it is integrated in the volume of V:

$$\begin{aligned}
\int_V \mathbf{s} : \nabla\mathbf{u}\, dV &= -\int_V \mathbf{u}\cdot(\nabla\cdot\mathbf{s})\, dV + \int_V \nabla\cdot(\mathbf{s}\cdot\mathbf{u})\, dV \\
&= -\int_V \mathbf{u}\cdot(\nabla\cdot\mathbf{s})\, dV - \int_S \mathbf{n}\cdot\mathbf{s}\cdot\mathbf{u}\, dS \\
&= -\int_V \mathbf{u}\cdot(\nabla\cdot\mathbf{s})\, dV - \int_S \mathbf{n}\cdot\mathbf{s}\cdot\mathbf{n}\phi\, dS \,.
\end{aligned} \tag{15}$$

Since $\mathbf{s}$ is a symmetric tensor defined by Eq. (6), the following identity also holds:

$$\mathbf{s} : \nabla\mathbf{u} = \mathbf{s} : (\nabla\mathbf{u})^T = \mathbf{s} : \mathbf{s} \,. \tag{16}$$

The local energy balance at the steady state can be obtained by taking the inner product of $\mathbf{u}$ and Eq. (5), and averaging it in time, as

$$0 = \nabla\cdot\left[-\frac{1}{2}\overline{(\mathbf{u}\cdot\mathbf{u})\mathbf{u}} - \overline{p\mathbf{u}}\right] + \frac{2}{Re_b}\overline{\mathbf{u}\cdot(\nabla\cdot\mathbf{s})} + \overline{\mathbf{u}\cdot\mathbf{b}} \,. \tag{17}$$

The global energy balance at the steady state can be obtained by integrating Eq. (17) in volume and by applying Eqs. (7), (14), (15), and (16), as

$$
\begin{aligned}
0 &= \int_V \nabla \cdot \left[-\frac{1}{2}\overline{(\mathbf{u}\cdot\mathbf{u})\mathbf{u}} - \overline{p\mathbf{u}} \right] dV + \frac{2}{Re_b}\int_V \overline{\mathbf{u}\cdot(\nabla\cdot\mathbf{s})}\, dV + \int_V \overline{\mathbf{u}\cdot\mathbf{b}}\, dV \\
&= W_p + \int_S \left[\frac{1}{2}\overline{(\mathbf{u}\cdot\mathbf{u})\mathbf{u}} + \overline{p\mathbf{u}} \right]\cdot\mathbf{n}\, dS - \frac{2}{Re_b}\left[\int_V \overline{\mathbf{s}:\mathbf{s}}\, dV - \int_S (\nabla\cdot\mathbf{n})\,\overline{\phi^2}\, dS \right] \\
&\quad + \int_V \overline{\mathbf{u}\cdot\mathbf{b}}\, dV \\
&= W_p + W_a - \frac{2}{Re_b}\int_V \overline{\mathbf{s}:\mathbf{s}}\, dV \ ,
\end{aligned}
\tag{18}
$$

where the pumping power, W_p, is expressed by

$$
W_p = \left(\int_A \overline{p\mathbf{u}}\cdot\mathbf{e}_1\, dA - \int_{A'} \overline{p\mathbf{u}}\cdot\mathbf{e}_1\, dA' \right) = AU_b\Delta P \tag{19}
$$

and the power required for actuation, W_a, is a summation of the power due to the blowing/suction from the wall (ϕ) and the body force ($\mathbf{b}$), i.e.,

$$
W_a = \int_S \left[\frac{1}{2}\overline{\phi^3} + \overline{p'\phi} + \frac{2}{Re_b}(\nabla\cdot\mathbf{n})\,\overline{\phi^2} \right] dS + \int_V \overline{\mathbf{u}\cdot\mathbf{b}}\, dV \ . \tag{20}
$$

The power due to the the wall-blowing/suction is composed of the pressure work (i.e., the first and second terms) and the additional work due to the wall-curvature (i.e., the third term): the former is identical to that presented in the previous studies applying blowing/suction in a plane channel [7, 17], while the latter is consistent with those presented in the studies accounting for deformable walls [18, 19].

By rearranging Eq. (19) and applying the Reynolds decomposition to $\overline{\mathbf{s}:\mathbf{s}}$, the global energy balance reads

$$
W_p + W_a = \frac{2}{Re_b}\int_V \bar{\mathbf{s}}:\bar{\mathbf{s}}\, dV + \frac{2}{Re_b}\int_V \overline{\mathbf{s}':\mathbf{s}'}\, dV \ . \tag{21}
$$

Equation (21) is essentially the same as the well-known relationship to the turbulent duct flows, except that W_a is added in the left-hand-side; the first and the second terms in the RHS of Eq. (21) represent the dissipation from the mean and fluctuation velocities, respectively.

In order to examine the lower bound of $W_p + W_a$, we focus on the first term in the RHS of Eq. (21). We decompose the mean velocity, $\overline{\mathbf{u}}$, into the Stokes flow solution, $\overline{\mathbf{u}}^S$ and the deviation therefrom, $\overline{\mathbf{u}}^D$, i.e.,

$$\overline{\mathbf{u}} = \overline{\mathbf{u}}^S + \overline{\mathbf{u}}^D \; . \tag{22}$$

Hereafter, the superscripts of S and D denote the quantities of the Stokes flow at the same bulk Reynolds number and the deviation therefrom, respectively. The Stokes part satisfies the continuity and the Stokes equation, i.e.,

$$\nabla \cdot \overline{\mathbf{u}}^S = 0 \; , \tag{23}$$

$$0 = \frac{2}{Re_b} \nabla \cdot \overline{\mathbf{s}}^S - \nabla \overline{p}^S \; , \tag{24}$$

and the no-slip wall boundary condition, i.e.,

$$\overline{\mathbf{u}}^S_{wall} = 0 \; . \tag{25}$$

Under the no-slip condition, the wall-normal component of $\overline{\mathbf{s}}^S$ is computed on the wall (see, Eq. (14)) as

$$\mathbf{n} \cdot \overline{\mathbf{s}}^S \cdot \mathbf{n} = 0 \; . \tag{26}$$

The deviation part should satisfy the continuity,

$$\nabla \cdot \overline{\mathbf{u}}^D = 0 \; . \tag{27}$$

and the blowing/suction wall boundary condition, i.e.,

$$\overline{\mathbf{u}}^D_{wall} = \overline{\phi} \, \mathbf{n} \; . \tag{28}$$

Moreover, due to the condition of constant flow rate, the flow rate due to the deviation part is zero, i.e.,

$$\int_A \overline{\mathbf{u}}^D \cdot \mathbf{e}_1 \, dA = 0 \; . \tag{29}$$

By using this velocity decomposition, the first term in RHS of Eq. (21) can also be decomposed as

$$\frac{2}{Re_b}\int_V \overline{\mathbf{s}}:\overline{\mathbf{s}}\,dV = \frac{2}{Re_b}\int_V \overline{\mathbf{s}}^S:\overline{\mathbf{s}}^S\,dV + \frac{4}{Re_b}\int_V \overline{\mathbf{s}}^S:\overline{\mathbf{s}}^D\,dV + \frac{2}{Re_b}\int_V \overline{\mathbf{s}}^D:\overline{\mathbf{s}}^D\,dV \ . \tag{30}$$

The second term in RHS of Eq. (30) can be rewritten by using Eqs. (15), (16), (24), (26)–(29) as

$$\begin{aligned}
\frac{4}{Re_b}\int_V \overline{\mathbf{s}}^S:\overline{\mathbf{s}}^D\,dV &= -\frac{4}{Re_b}\int_V \overline{\mathbf{u}}^D\cdot(\nabla\cdot\overline{\mathbf{s}}^S)\,dV - \frac{4}{Re_b}\int_S \underbrace{\mathbf{n}\cdot\overline{\mathbf{s}}^S\cdot\mathbf{n}}_{=0}\overline{\phi}\,dS \\
&= -2\int_V \overline{\mathbf{u}}^D\cdot\nabla\overline{p}^S\,dV \\
&= -2\int_V \left[\nabla\cdot(\overline{p}^S\overline{\mathbf{u}}^D) - \overline{p}^S\underbrace{\nabla\cdot\overline{\mathbf{u}}^D}_{=0}\right]dV \\
&= 2\left[\int_A \overline{p}^S\overline{\mathbf{u}}^D\cdot\mathbf{e}_1\,dA - \int_{A'} \overline{p}^S\overline{\mathbf{u}}^D\cdot\mathbf{e}_1\,dA'\right] + 2\int_S \overline{p}^S\overline{\mathbf{u}}^D\cdot\mathbf{n}\,dS \\
&= 2\Delta P^S\underbrace{\int_A \overline{\mathbf{u}}^D\cdot\mathbf{e}_1\,dA}_{=0} + 2\int_S \overline{p}^S\overline{\phi}\,dS \\
&= 2\int_S \overline{p}^S\overline{\phi}\,dS \ .
\end{aligned} \tag{31}$$

By substituting Eqs. (30) and (31) into Eq. (21), we obtain the final expression for the energy balance:

$$\begin{aligned}
W_p + W_a = \underbrace{\frac{2}{Re_b}\int_V \overline{\mathbf{s}}^S:\overline{\mathbf{s}}^S\,dV}_{\text{(I)}} + \underbrace{\frac{2}{Re_b}\int_V \overline{\mathbf{s}}^D:\overline{\mathbf{s}}^D\,dV}_{\text{(II)}} + \underbrace{\frac{2}{Re_b}\int_V \overline{\mathbf{s}':\mathbf{s}'}\,dV}_{\text{(III)}} \\
+ \underbrace{2\int_S \overline{p}^S\overline{\phi}\,dS}_{\text{(IV)}} \ .
\end{aligned} \tag{32}$$

The first term in RHS (denoted as (I)) is the dissipation rate of the Stokes flow. For a cylindrical pipe and a plane channel, term (I) can be expressed simply as $2/Re_b$ and $3/(4Re_b)$, respectively (where the unit length (δ^*) is taken to be the pipe radius/channel half-width). The second term (II) is the dissipation due to the deviation of mean velocity from the Stokes flow profile. The third term (III) is the dissipation from the fluctuating velocities. The fourth term (IV) may formally be interpreted as a correlation between the time-averaged blowing suction in the Navier-Stokes flow and the wall pressure in the corresponding Stokes flow, of which implication will be discussed in §4.

3 Lower bound of the net driving power for a controlled flow in a duct with constant-shape cross-section

The lower bound of the net driving power, $W_p + W_a$, for the flows in a duct with constant-shape cross-section (including a plane channel, a cylindrical pipe, and ducts with constant-shape cross-section, as shown in Fig. 1) can be derived as special cases of Eq. (32). In the Stokes flow in those geometries, the pressure gradient is constant and the pressure is uniform in the cross-sections normal to the flow direction, $\mathbf{e}_1$, i.e.,

$$-\nabla\overline{p}^S = \frac{\Delta P^S}{L}\mathbf{e}_1 \,, \tag{33}$$

where L is the periodic length ($L = V/A$ for the straight ducts, while it is a function of cross-sectional position for the ducts with constant streamline curvature). Accordingly, Eq. (31) is modified to read

$$\begin{aligned} \frac{4}{Re_b}\int_V \overline{\mathbf{s}}^S : \overline{\mathbf{s}}^D \, dV &= -2\int_V \overline{\mathbf{u}}^D \cdot \nabla\overline{p}^S \, dV \\ &= 2\Delta P^S \underbrace{\int_A \overline{\mathbf{u}}^D \cdot \mathbf{e}_1 \, dA}_{=0} \\ &= 0 \,. \end{aligned} \tag{34}$$

Therefore, term (IV) in Eq. (32) vanishes and the energy balance equation reduces to read

$$W_p + W_a = \underbrace{\frac{2}{Re_b}\int_V \overline{\mathbf{s}}^S : \overline{\mathbf{s}}^S \, dV}_{\text{(I)}} + \underbrace{\frac{2}{Re_b}\int_V \overline{\mathbf{s}}^D : \overline{\mathbf{s}}^D \, dV}_{\text{(II)}} + \underbrace{\frac{2}{Re_b}\int_V \overline{\mathbf{s}' : \mathbf{s}'} \, dV}_{\text{(III)}} \; . \tag{35}$$

Since terms (II) and (III) are non-negative, the lower bound of the total driving power is exactly the dissipation rate of the Stokes flow at the same bulk Reynolds number. This minimum power is achieved when the velocity profile is that of the Stokes flow. In other words, any control input that modifies the velocity profile from the Stokes flow always results in a larger net power. Of course, this argument holds also for the case of sublaminar drag where W_p is less than term (I). In that case, the actuation power, W_a, should always be larger than the summation of terms (II) and (III).

4 Discussion

Finally, we discuss two issues related to the energy balance for the ducts with variable cross-section, given by Eq. (32). First, if we impose the blowing/suction boundary condition to the Stokes part instead of the deviation part, i.e., $\overline{\mathbf{u}}^S_{wall} = \mathbf{n}\phi$ and $\overline{\mathbf{u}}^D_{wall} = 0$, the resulting energy balance becomes identical to Eq. (35). This result implies that the lower bound of the net power for a duct with variable cross-section can be given by the pumping power of the Stokes flow with a certain blowing/suction (of which distribution cannot be specified from the present analysis). Second, Eq. (32) implies a possibility to reduce the net power to a value below the pumping power of the uncontrolled Stokes flow if term (IV) can be made negative. Such a situation is conjectured to happen, for example, in a channel with a bump. Namely, if we apply a constant suction ($\overline{\phi} < 0$) in the front part of the bump where $\overline{p}^S$ is higher and a constant blowing ($\overline{\phi} > 0$) in the rear part of the bump where $\overline{p}^S$ is lower, then term (IV) might become negative. (this operation may be similar to modifying the streamlines so that they become closer to those in a plane channel without the bump). This last conjecture should be examined as the future direction in conjunction with the lower bound of net energy for external flows, which has not been clarified, either [17].

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