Adjoint Analysis of Heat and Fluid Flow Toward Optimal Shape Design of Recuperators*

Kenichi MORIMOTO†, Yuji SUZUKI† and Nobuhide KASAGI†

Key Words: *Recuperator, Adjoint Analysis, Optimal Shape Design, Heat Transfer Enhancement*

1. Introduction

Recently, growing attention has been paid to small-scale distributed energy systems with micro gas turbines in view of high efficiency and low environmental impact. It is known that one of the most important technical issues is improvement of recuperator effectiveness [1].

Among various compact heat exchangers, primary surface recuperators have been considered promising for recuperated turbine systems [2]. We have demonstrated that the proper use of oblique wavy walls in counter-flow recuperators could achieve significant heat transfer enhancement with relatively small pressure loss penalty [3].

In the present study, an optimal shape design method with adjoint variables of the velocity and thermal fields is employed in order to simultaneously optimize heat transfer and pressure loss characteristics. We describe the formulation and discuss the validity of this method.

2. Numerical Procedure

Figure 1 shows the passage geometry and coordinate system employed in the present study. The initial surface shape of the top and bottom walls is given by the following equation [3]:

$$y_{w,\,top} = y_{w,\,bottom} = -A\cos 2\pi\big/L_x\left(x - z\tan\gamma\right), \qquad (1)$$

where L_x and $\gamma\left(=\tan^{-1}\left(L_x/2\delta\right)\right)$ denote the streamwise pitch and oblique angle of wavy surface, respectively. The working fluid is air and the bulk mean temperature is kept constant at the inlet. The Reynolds number Re based on the bulk mean streamwise velocity U_b and the duct half-height δ is set constant at 100. A periodic boundary condition is imposed in the streamwise (x-) direction. As a thermal boundary condition, isothermal heating is assumed on the four walls.

3. Optimal Shape Design Procedure

Figure 2 shows a schematic diagram of the shape optimization. The present simulation is performed under the following assumptions:

1) The top and bottom walls are movable, while the left and right walls are kept always flat in shape and fixed in position.
2) The streamwise length L_x is fixed.
3) The bulk mean velocity is kept constant.
4) The volume of the duct is kept constant by adjusting the mean distance between the top and bottom walls L_y.

The distribution of the shape modification $\rho\left(\boldsymbol{x}_\Gamma\right)$ is determined by almost the same procedure proposed by Çabuk and Modi (1992), while the energy equation is additionally incorporated into the optimization procedure in order to take into account the heat transfer characteristics. The cost function J to be maximized is defined as follows:

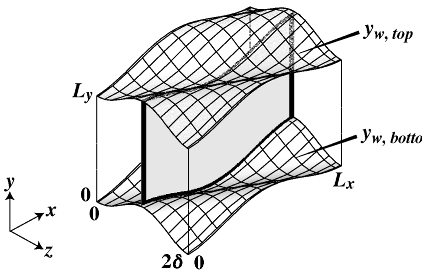

Fig.1 Heat-exchanger passage with oblique wavy walls.

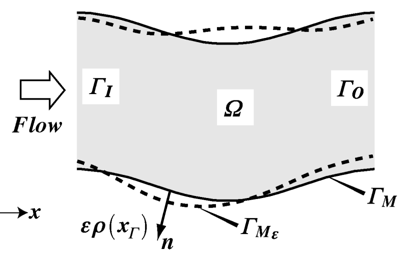

Fig. 2 Schematic of the shape optimization.

$$J\left(\Gamma_M\right) = \int_{\Gamma_I+\Gamma_O} p\, d\Gamma\Big/P_0 + \beta\int_{\Gamma_M}\frac{1}{Pe}\frac{\partial T}{\partial n}\,d\Gamma\Big/Q_0, \qquad (2)$$

where the first and second terms respectively represent the mean pressure loss between the inlet and outlet boundaries, and heat transfer integrated over the entire walls. Each term is normalized with its absolute quantity in the equivalent straight square duct denoted by P_0 and Q_0, respectively. In Eq. (2), β represents the weighting factor of heat transfer to pressure loss, and is set to unity in the present simulation.

The Navier-Stokes and energy equations are

$$u_{i,i} = 0, \quad u_j u_{i,j} + p_{,i} - \frac{1}{Re}u_{i,jj} = 0,$$
$$u_j T_{,j} - \frac{1}{Pe}T_{,jj} = 0 \quad \text{in } \Omega, \qquad (3)$$

where the velocity and pressure are nondimensionalized by U_b, δ, and ν with T being the temperature difference from the wall temperature T_w.

Let $\left(p',\, u_i',\, T'\right)$ be respectively the variations of $\left(p,\, u_i,\, T\right)$ in response to the shape modification of the passage from Γ_M to $\Gamma_{M\varepsilon}$. Since the velocity and thermal fields satisfy the non-slip and isothermal wall condition on Γ_M, respectively, the following relations can be derived by using Taylor series expansions:

$$u_i' = -\rho\frac{\partial u_i}{\partial n} \quad \text{and} \quad T' = -\rho\frac{\partial T}{\partial n} \quad \text{on } \Gamma_M. \qquad (4)$$

* Received : July 9, 2005, Editor : Shigefumi NISHIO

† Department of Mechanical Engineering, The University of Tokyo (Hongo 7-3-1, Bunkyo-ku, Tokyo 113-8656, JAPAN)

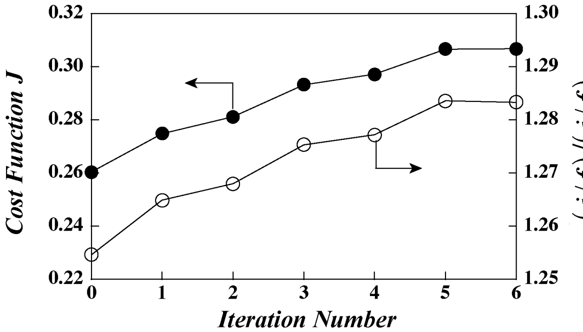

Fig. 3 History of the cost function and j/f factor.

Now, we introduce the adjoint variables $\left(p^*,\ u_i^*,\ T^*\right)$, which satisfy the following adjoint equations:

$$u^*_{i,i} = 0,\quad u_j\left(u^*_{i,j} + u^*_{j,i}\right) - p^*_{,i} + \frac{1}{Re}u^*_{i,jj} - TT^*_{,i} = 0,$$
$$-u_jT^*_{,j} - \frac{1}{Pe}T^*_{,jj} = 0 \quad \text{in } \Omega. \tag{5}$$

Then, by using the divergence theorem, the first variation of the cost function δJ can be derived as:

$$\delta J = \frac{1}{P_0}\int_{\Gamma_M}\rho\left(\frac{1}{Re}\frac{\partial u_i}{\partial n}\frac{\partial u_i^*}{\partial n} - \frac{1}{Pe}\frac{\partial T}{\partial n}\frac{\partial T^*}{\partial n}\right)d\Gamma, \tag{6}$$

where the following boundary conditions are imposed on the adjoint variables:

$$u_i^* = 0 \text{ and } T^* = \frac{P_0}{Q_0}\beta\ (= Const.) \text{ on } \Gamma_M. \tag{7}$$

At the inlet and outlet boundaries, a periodic boundary condition is imposed, instead of the inflow and outflow conditions used in the procedure of Çabuk and Modi (1992).

We employ a steepest gradient method, and determine $\rho\left(\boldsymbol{x}_\Gamma\right)$ as:

$$\rho\left(\boldsymbol{x}_\Gamma\right) = \frac{1}{Re}\frac{\partial u_i}{\partial n}\frac{\partial u_i^*}{\partial n} - \frac{1}{Pe}\frac{\partial T}{\partial n}\frac{\partial T^*}{\partial n}. \tag{8}$$

For the ease of the grid generation, each grid point is assumed movable only in the y-direction, while fixed in the x- and z- directions. To do this, the wall-normal deformation expressed by Eq. (8) is transformed to the movement in the y-direction. In this study, ε is set constant such that the magnitude of the wall deformation is kept less than 0.05δ in order to avoid numerical instability.

4. Results of the Optimization

The initial shape is chosen as $A = 0.25\delta$ and $\gamma = 60°$ in Eq. (1), with which the maximum j/f factor is obtained [3].

Figure 3 shows the history of the cost function J and the corresponding change of the j/f factor. The cost function increases monotonically with successive iterations and reaches its maximum at the 5th iteration. The j/f factor takes its maximum value at the 5th iteration as well, and about 3% improvement is achieved. Whereas the pressure loss fRe is almost the same as the initial case, the contribution of the skin friction is increased. Therefore, it is conjectured that the increase in the averaged Nusselt number is due to the reduction of the separation bubble and thus the enhancement of heat transfer associated with the wall shear flow. It is noted that, after the 7th iteration, computation diverges due to the appearance of locally-steep surfaces unresolvable with the present grid system.

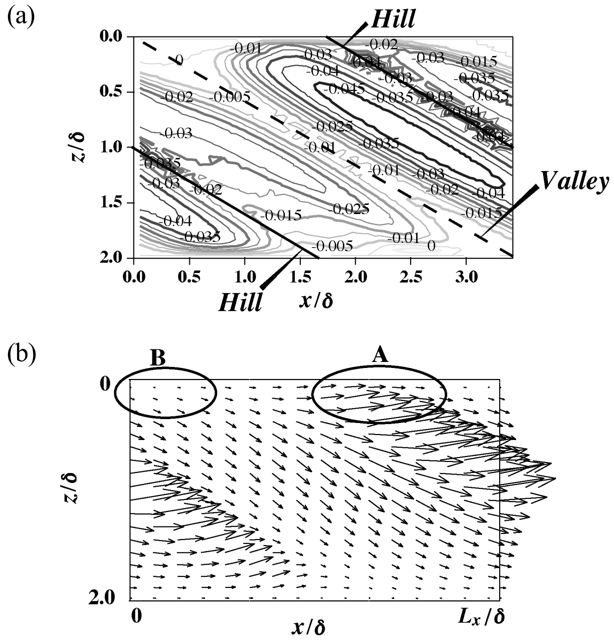

Fig. 4 Modified structure at the 5th iteration: (a) Cumulative surface deformation from the initial shape to the 5th step on the bottom wall, (b) Wall shear stress vectors on the bottom wall.

Figure 4 shows cumulative surface deformation from the initial state to the 5th step, and the wall shear stress vectors on the bottom wall at the 5th iteration. It can be seen that there is large negative deformation on both sides of the hill region. Thus, the shape of the hill becomes steeper than the initial shape, and the valley region becomes leveled. With this modification, the flow over the hill near the left wall (A) is kept attached to the wall and the flow separation region of the valley (B) in the initial case is diminished. Therefore, in the neighborhood of the initial shape with oblique wavy walls presently employed, the maximum j/f factor can be achieved by enhancing the wall shear flow and suppressing the flow separation simultaneously.

5. Concluding Remarks

The optimal shape design procedure is established using a cost function, which represents the weighted mean of the heat transfer and pressure loss characteristics. It is shown in the present simulation that the j/f factor can be further increased by the shape optimization method developed.

Acknowledgement

This work was supported through the Grant-in-Aid for JSPS Fellows (No.15–11654) by the Ministry of Education, Culture, Sports, Science and Technology of Japan (MEXT).

References

[1] H. Uechi, S. Kimijima, and N. Kasagi, ASME J. Eng. Gas Turbines Power **126**, 755 (2004).
[2] C. F. McDonald, Appl. Therm. Eng. **20**, 471 (2000).
[3] K. Morimoto, Y. Suzuki, and N. Kasagi, *Proc. 6th KSME-JSME Therm. Fluids. Eng. Conf.*, Paper DD.02. (2005).
[4] H. Çabuk, and V. Modi, J. Fluid Mech. **237**, 373 (1992).